

Hence

$$\begin{aligned} & \{\Pi(1,p) - \Pi(-1,p)\} - \{\Pi(1,p-1) - \Pi(-1,p-1)\} \\ &= n_a + n_b - n_e - n_f - n_d - n_e + n_b + n_c \\ &= (n_a + n_c + n_e) - (n_b + n_d + n_f) \pmod{3} \\ &= \{\Pi(1,1) - \Pi(-1,1)\} \pmod{3}. \end{aligned}$$

This is just another way of expressing formula (5), since hexagonal and trigonal SiC structures (space groups $P6_3mc$ and $P3m1$) are characterized by $\{\Pi(1,1) - \Pi(-1,1)\} = 0 \pmod{3}$, and rhombohedral structures ($R3m$) by $\{\Pi(1,1) - \Pi(-1,1)\} = \pm 1 \pmod{3}$.

3). The positive sign refers to the obverse setting, the negative sign to the reverse one.

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The Crystal Structure of Lanthanum Rhenium Oxide, $\text{La}_4\text{Re}_6\text{O}_{19}$

BY NORMAN L. MORROW* AND LEWIS KATZ

Department of Chemistry and Institute of Materials Science, University of Connecticut, Storrs, Connecticut, U.S.A.

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Lanthanum rhenium oxide, $\text{La}_4\text{Re}_6\text{O}_{19}$, is a body centered cubic phase, $a = 9.0325 \pm 0.0014 \text{ \AA}$, space group $I23$, $Z = 2$. It is formed by reacting La_2O_3 , ReO_3 , and Re at 1000° in vacuum. The structure contains a three-dimensional network of ReO_6 octahedra in which pairs of octahedra share an edge; the pairs in turn are linked to each other through corner sharing. The Re-Re distance in each pair is 2.42 \AA , indicating metal-metal bonding. The Re-O distances range from 1.98 – 2.01 \AA . The lanthanums are present as tetrahedral La_4O groups in open spaces of the network. The lanthanum coordination number is 10 with La-O distances of 2.50 \AA (1); 2.51 \AA (3); 2.60 \AA (3); and 2.88 \AA (3).

Introduction

If lanthanum sesquioxide, rhenium trioxide, and rhenium metal, in proportions corresponding approximately to $\text{La}_2\text{Re}_2\text{O}_7$, are heated to 1000° in vacuum, a heterogeneous product is formed (Longo, 1964). The major component of this product is a body centered cubic phase. No technique was found to separate this phase from the others present, so the stoichiometry could not be determined reliably in the normal way from density and chemical analysis. Microscopic examination of the material showed crystals apparently suitable for single-crystal study, so it was decided to solve the structure and from it establish the composition of the cubic phase.

Experimental

Longo (1964) obtained a good powder pattern for the cubic lanthanum rhenium oxide, which was indexed using a body centered cell with $a = 9.05 \text{ \AA}$. A more accurate cell length was obtained by Cohen's least-squares method (Klug & Alexander, 1954), using a

program of Leipold & Pauly (1963). The value for a obtained is $9.0325 \pm 14 \text{ \AA}$.

Under the microscope a number of black (or dark blue) crystals were observed. Their habit was multifaceted with one large face—apparently the face which had been in contact with the walls of the capsule—so that the crystals roughly approximated hemispheres. The radii of these 'hemispheres' were in the range 0.05 – 0.20 mm . The crystal which was selected had base diameters ranging from 0.130 to 0.165 mm and a height of 0.135 mm .

A precession camera study revealed directions with C_6 zero level symmetry and C_3 upper level symmetry as well as directions showing C_{2l} symmetry for both zero and upper levels. This establishes Laue group $m\bar{3}$ (Buerger, 1942). All of the photographs showed absences for odd values of $h+k+l$. The probable space group is therefore one of $I23$, $I2_13$ and $Im\bar{3}$.

The crystal was aligned with the c axis vertical when $\chi = 0$ on a General Electric single crystal orienter. Zr-filtered Mo radiation was used with pulse height discrimination, a scintillation counter, and a 2° take-off angle to align the crystal and collect data. A total of 2634 reflections were measured by the moving-crystal moving-counter technique. Background was measured on each side (in θ) of each reflection. Averaging equiv-

* Present address: Analytical Research Division, Esso Research and Engineering Company, Linden, New Jersey.

alent reflections gave a total of 772 independent reflections. Using an approximate density and cell content, a linear absorption coefficient of 600 cm^{-1} was estimated. (The actual value, $\mu = 650 \text{ cm}^{-1}$, was used in the final refinement.) Because of the large absorption coefficient of the material, an unusual collection scheme was employed. All accessible reflections for which $l \leq 3$ and all their higher orders were measured. For this crystal and orientation, almost all of the collected reflections occur at low values of χ ($< 20^\circ$) and in a small range of φ (about 30°). The need for an absorption correction dependent on χ and φ is thus reduced.

The data were corrected for Lorentz and polarization effects, and a sharpened three-dimensional Patterson map was generated using programs of Penfold (1959) and Sly, Shoemaker & Van den Hende (1962). The Patterson map was consistent with rhenium atoms in positions 12(e) of space groups $I23$ and $Im\bar{3}$ with x approximately $\frac{1}{8}$ or $\frac{3}{8}$, but was not consistent with any position of $I2_3$. To resolve the ambiguity in x , two cycles of isotropic least-squares calculation (Busing & Levy, 1959) were run; only the value $x = \frac{3}{8}$ refined. With the Re atoms located, the Re-La peaks on the Patterson were identified. These indicated either position 8(c) of $I23$ or 16(f) of $Im\bar{3}$ for lanthanum with $x = \frac{1}{6}$ or $\frac{2}{6}$. However, the preparations indicated that the number of lanthanum atoms could not exceed the number of rhenium atoms and was probably smaller, so 8(c) of $I23$ was chosen. $I23$ was confirmed as the proper space group by the structure determination. It was impossible to decide at this stage between $x = \frac{1}{6}$ and $x = \frac{2}{6}$ because the only difference is the sign of the imaginary part of the structure factor; *i.e.* the two values correspond to two enantiomorphs. The $x = \frac{1}{6}$ value was arbitrarily chosen.

Least-squares refinement of the rhenium and lanthanum positions gave an unreasonably high R value, though the convergence was regular. A spherical absorption correction of the data was made with $\mu R = 4.3$. Refinement of the corrected data gave an R value of 12.4%.

Using the corrected data and the refined rhenium and lanthanum parameters, a difference electron density map was calculated. Three crystallographically different sets of oxygen atom positions were located. These positions are 2(a), 12(d), and 24(f). Thus the total unit cell formula for the material was established as $\text{La}_8\text{Re}_{12}\text{O}_{38}$. The density calculated from this formula, 8.91 g.cm^{-3} , agrees well with the observed density, $8.92 \pm 0.06 \text{ g.cm}^{-3}$. Introduction of the oxygen atoms into the least-squares refinement, treating temperature factors anisotropically, led to an R value of 7.4%.

The ten largest reflections showed obvious extinction effects. Omitting these and the next five largest reflections and introducing the weighting scheme $|w| = 1$ for $|F|_{\text{obs}} > 60$ and $|w| = |F|_{\text{obs}}/60$ for $|F|_{\text{obs}} \leq 60$ led to an R value of 6.6% (weighted $R = 6.6\%$ also) as the final result. The scattering factors used throughout were those of Re^{4+} and La^{3+} (Thomas & Umeda, 1957) and O^{2-} (*International Tables*, 1962). Both the real and imaginary anomalous dispersion corrections were made for rhenium and lanthanum (Cromer, 1965). All refinements were based on F .

To confirm the stoichiometry another difference map was calculated using the final parameters. It showed no peaks of height greater than one-fifth of an oxygen. In addition a least-squares refinement was carried out with the atom multipliers, except for rhenium and the twofold-axis oxygen, as variables. No significant changes from full occupancy occurred. The atomic position and thermal parameters are given in Table 1, and the corresponding structure factors are given in Table 2.

The data are reported with only positive values of h , k , and l . In general these are averages of measurements of hkl and $\bar{h}\bar{k}l$ reflections. Such reflections may properly be averaged since they are related by genuine twofold symmetry. The Friedel related reflections were not recorded. Effectively, the one octant of data listed represents half of the available sphere in reciprocal space: $hkl, \bar{h}\bar{k}l, h\bar{k}l, \bar{h}kl$. Since the structure does not have a center of symmetry, it remained to be deter-

Table 1. Atomic parameters

Position parameters $\times 10^5$ (standard errors in parentheses)

	x	y	z
Re	36632 (7)	$\frac{1}{2}$	0
La	15986 (9)	15986 (9)	15986 (9)
O(1)	32247 (155)	0	0
O(2)	34136 (130)	28390 (113)	97389 (122)
O(3)	0	0	0

The temperature factor expression used was: $\exp [-(h^2\beta_{11} + k^2\beta_{22} + l^2\beta_{33} + 2hk\beta_{12} + 2hl\beta_{13} + 2kl\beta_{23})]$.Thermal parameters $\times 10^5$ (standard errors in parentheses)

	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
Re	76 (4)	82 (4)	82 (4)	0	0	3 (7)
La	165 (4)	165 (4)	165 (4)	15 (5)	15 (5)	15 (5)
O(1)	144 (79)	120 (69)	228 (91)	0	0	-37 (139)
O(2)	205 (70)	119 (57)	194 (67)	31 (66)	159 (75)	21 (59)
O(3)	236 (130)	236 (130)	236 (130)	0	0	0

Table 2. Observed and calculated structure factors
 Asterisks mark the reflections omitted from the final refinement.

Table with multiple columns (A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z) containing numerical data for observed and calculated structure factors. Includes sub-headers like 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J', 'K', 'L', 'M', 'N', 'O', 'P', 'Q', 'R', 'S', 'T', 'U', 'V', 'W', 'X', 'Y', 'Z' and various numerical values.

mined whether for the model presented the recorded hkl should have been $\bar{h}\bar{k}\bar{l}$, or, alternatively, whether for the indices recorded the correct model should have been the center related one. To test the compatibility of the model and the indexing two new least-squares refinements were kindly carried out for us by Dr Arthur Sleight, one refinement with $\Delta f''$ and the other with $-\Delta f''$. This change of sign is equivalent to changing the signs of the indices or of changing the signs of all the position parameters in so far as the magnitudes of the structure factors are concerned. The results made it clear that our original assignment was correct. Using $\Delta f''$, R dropped to 69%, whereas with $-\Delta f''$ the lowest value for R was 85%. In addition, all the errors were larger with the changed signs of $\Delta f''$. The large values for $\Delta f''$ for Re (7.34) and for La (2.86) made it fairly easy to resolve the issue. Certainly the difference in R was more pronounced than for some cases already reported (Hamilton, 1965; Veki, Zalkin & Templeton, 1966; Zalkin, Hopkins & Templeton, 1966).

Since the structure belongs to the non-polar class 23, the anomalous atoms should not be misplaced relative to the others in the case of the refinement to the false minimum (Cruickshank & McDonald, 1967). Though there were differences in the final atomic positions for the two refinements, these differences were indeed small; all were less than their corresponding standard deviations. (Since 23 is an enantiomorphous class, the refinement with $-\Delta f''$ corresponds to the opposite absolute configuration to the true structure.)

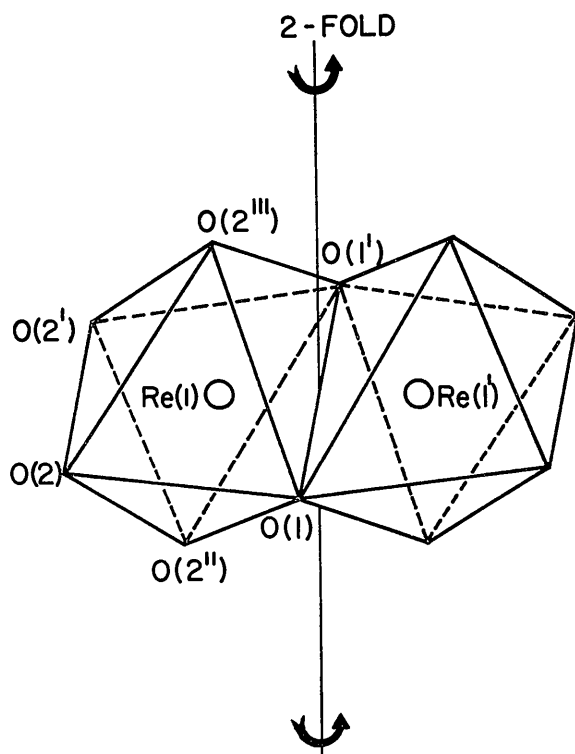


Fig. 1. Pair of edge sharing ReO_6 octahedra.

Discussion

A basic feature of the structure of $\text{La}_4\text{Re}_6\text{O}_{19}$ is the highly symmetric three-dimensional network of ReO_6 octahedra. The octahedra occur in edge-shared pairs; one such pair is shown in Fig. 1. These octahedra are quite distorted, as indicated by the distances and angles listed in Table 3. The rhenium atoms in the pairs are only 2.42 Å apart, which is appreciably closer than the distance in rhenium metal (about 2.75 Å), and is intermediate between the very short (2.24 Å) distance found in the $[\text{Re}_2\text{Cl}_8]^{2-}$ ion and the average Re-Re distance of 2.48 Å found in Re_3Cl_9 groups in Re(III) compounds derived from rhenium(III) chloride and in rhenium(III) chloride itself (Cotton & Harris, 1965).^{*} Despite this

Table 3. *Interatomic distances and angles**

Re-Re (in edge-shared octahedra)	2.415 (1) Å
	3.522 (1)
La-La (in La_4O tetrahedra)	4.084 (2)
	5.070 (1)
Re-La	3.403 (1)
Rhenium coordination (Fig. 1)	
Re—O(1), O(1')	2.007 (10) Å
Re—O(2), O(2')	1.979 (9)
Re—O(2''), O(2''')	1.987 (10)
O(1)—O(1')	3.207 (26)
O(2)—O(2')	2.903 (22)
O(1)—O(2); O(1')—O(2')	2.581 (9)
O(1)—O(2''); O(1')—O(2''')	2.781 (11)
O(1)—O(2''); O(1')—O(2''')	3.040 (13)
O(2)—O(2''); O(2')—O(2''')	2.738 (5)
O(2)—O(2''); O(2')—O(2''')	2.647 (3)
O(1)—Re—O(1')	106.0 (4)°
O(2)—Re—O(2')	93.9 (6)
O(1)—Re—O(2); O(1')—Re—O(2')	80.5 (4)
O(1)—Re—O(2''); O(1')—Re—O(2''')	88.5 (3)
O(1)—Re—O(2''); O(1')—Re—O(2''')	99.4 (3)
O(2)—Re—O(2''); O(2')—Re—O(2''')	87.3 (2)
O(2)—Re—O(2''); O(2')—Re—O(2''')	83.8 (2)
O(2'')—Re—O(2''')	167.0 (6)
O(1)—Re—O(2); O(1')—Re—O(2')	170.6 (3)
Lanthanum coordination (Fig. 3)	
La—O(1) O(1'') O(1''')	2.515 (7) Å
La—O(2) O(2'v) O(2'v')	2.601 (9)
La—O(2'') O(2'vi) O(2'vii)	2.882 (10)
La—O(3)	2.501 (1)
O(1)—O(2); O(1'')—O(2'v); O(1''')—O(2'v')	2.581 (9)
O(1)—O(2''); O(1'')—O(2'vii); O(1''')—O(2'vi)	2.781 (11)
O(1)—O(2'v); O(1'')—O(2'vi); O(1''')—O(2)	3.113 (11)
O(2)—O(2''); O(2'iv)—O(2'vi); O(2'v)—O(2'vii)	2.738 (5)
O(2'')—O(2'vi); O(2'vi)—O(2'vii); O(2'vii)—O(2'')	3.716 (20)
O(2)—O(2'vi); O(2'')—O(2'v); O(2'iv)—O(2'vii)	2.647 (3)
O(3)—O(1); O(1''); O(1''')	2.913 (13)

* Calculated using the program of Busing, Martin & Levy (1964). Parenthesized quantities are standard deviations.

* An Re-Re distance of 2.48 Å is also reported by Magnéli (1957) in the monoclinic modification of ReO_2 . The accuracy of this value is not known, however, since the Re positions were simply assumed to be the same as the Mo positions in the corresponding modification of MoO_2 . For the Re-Re distance in orthorhombic ReO_2 , Magnéli (1957) reports a value of 2.61 Å. In this case the Re positions were determined from Harker lines only, and again no estimate of accuracy is given.

close Re-Re approach, the distortion of the octahedra results in nearly equal Re-O distances.

A representation of the three-dimensional network is shown in Fig. 2. In part (a), to emphasize the pattern of this network the octahedra have been replaced by rectangular parallelepipeds. The unit cell is outlined in broken lines. In part (b), the octahedra of the front (and back) face are shown. In part (c), the octahedra of the center section are shown. Large openings in the network of ReO_6 octahedra are clearly revealed at the

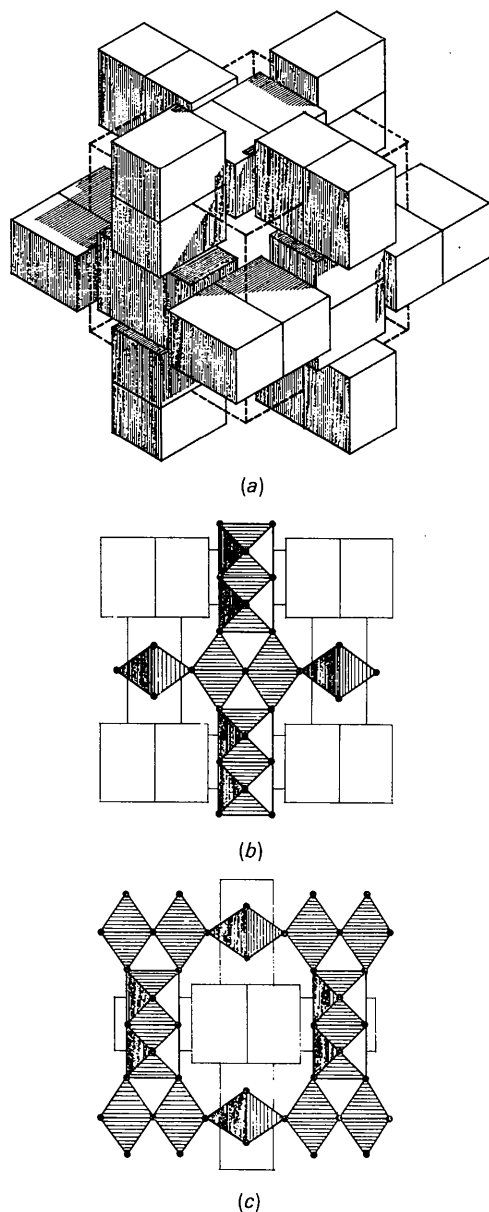


Fig. 2. Representation of rhenium-oxygen network. In part (a) the three-dimensional network is shown with parallelepipeds representing inscribed octahedra. The unit cell is outlined with broken lines. In (b) the front (and back) section is shown as connected octahedra. In (c) the center section is shown.

corners and body center of the cell. This same type of network has been reported for a high temperature form of KSbO_3 (Spiegelberg, 1940) and in KBiO_3 (Zemann, 1950). These structures were determined from powder data, and the regularity of the octahedra is unknown.

In the holes in the network there are tetrahedral La_4O groups. Each group is a perfect tetrahedron of lanthanums with an oxygen at its center. The lanthanum-oxygen distance is $2.501 \pm 10 \text{ \AA}$ and the lanthanum-lanthanum distance is $4.084 \pm 2 \text{ \AA}$. Since the tetrahedra surround lattice points, the tetrahedra are identical and have the same orientation.

The lanthanum environment is a modification of the seven-coordination found in lanthanum sesquioxide (Pauling, 1928; Koehler & Wollan, 1953). In $\text{La}_4\text{Re}_6\text{O}_{19}$, the sevenfold coordination is modified to add three additional oxygens so that the coordination is tenfold. An alternative description would be to say that the coordination polyhedron is a trigonal prism capped on four faces but not on one of the triangular faces. The lanthanum-oxygen bond distances are listed in Table 3, and the lanthanum coordination polyhedron is shown in Fig. 3.

The formal oxidation number for rhenium in $\text{La}_4\text{Re}_6\text{O}_{19}$ is $4\frac{1}{3}$. This suggested the possibility of interesting magnetic and electrical properties. These properties have in fact been studied and will be reported elsewhere, together with a detailed structure determination done from powder data, by Drs J. M. Longo and A. W. Sleight. This structure determination (Sleight & Longo, 1967) was carried out independently but agrees well with our single crystal results.

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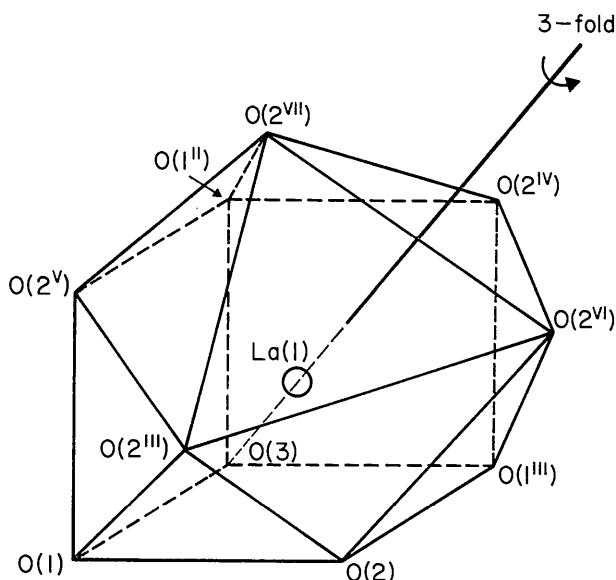


Fig. 3. Lanthanum environment.

through grant GP-5934 and by the University of Connecticut Research Foundation. Computations were carried out in the Computer Center of the University of Connecticut, which is supported in part by grant GP-1819 of the National Science Foundation. We are grateful to Dr A. W. Sleight for carrying out the absolute configuration test while our Computer Center was replacing its main computer.

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The Conformation of Non-Aromatic Ring Compounds XLVI.* The Crystal and Molecular Structure of 2,2'-Dichlorotrimethylene Sulphite

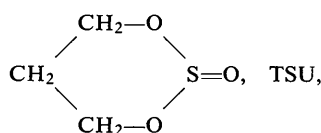
BY JOHANNA W. L. VAN OYEN, R. C. D. E. HASEKAMP, G. C. VERSCHOOR AND C. ROMERS
Organic Chemistry Department, University of Leiden, The Netherlands

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2,2'-Dichlorotrimethylene sulphite crystallizes in the form of monoclinic crystals with space group $P2_1$ and $Z=2$. The unit cell dimensions are $a=6.173$, $b=6.500$, $c=8.986$ Å and $\beta=104.7^\circ$, at 20°C . The diffraction data were collected at room temperature with a diffractometer using Mo $K\alpha$ radiation. The structure was solved by the heavy-atom technique and refined by the least-squares method, the final R value being 5.0%. The molecule has the chair conformation with the S=O group pointing towards the axial direction. The conformation features are discussed and compared with those of analogous compounds.

Introduction

During the past decade the conformation of cyclic sulphites, notably trimethylene sulphite



has been frequently investigated. They have been mainly studied in the liquid phase by the application of a number of physical methods such as the infrared technique, nuclear magnetic resonance and electric dipole measurements. The interpretation of the physical properties was, however, difficult and resulted in conflicting conclusions concerning the conformation. The papers of van Woerden & Havinga (1967*a,b*) give details and further references.

Altona, Geise & Romers (1966) determined the crystal structure of TSU at -100°C and proved that in the solid state it has the chair conformation with

* Part XLV, Altona, Hageman & Havinga, (1968).